AD-A277 377

IMENTATION PAGE

. REPORT DATE	3. REPORT TYPE AND DATES COVERED
1/10/94	Final Technical (05/01/91 - 8/31/93)

4. TITLE AND SUBTITLE

Modeling and Control of Nonlinear Systems

5. FUNDING NUMBERS

AFOSR-91-0255

6. AUTHOR(S)

Professor John E. Hauser

94-09046

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

University of Southern California
Department of EE-Systems
Los Angeles, CA 90089-2563

AFOSR-TR- 94 0052

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Marc Q. Jacobs
AFOSR/NM
110 Duncan Ave., Suite B115
Bolling AFB, DC 20332-0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

DTIC
ELECTE
MAR 23 1994
S F D

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution unlimited.

Approved for public release;
distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

We have conducted research on the modeling and control of nonlinear systems. Our efforts have been directed toward understanding the control of truly nonlinear behavior as well as the synthesis of control laws for systems that can be nearly transformed into linear systems (via approximate feedback linearization).

We have introduced a new framework for understanding and analyzing the stability and control of nonlinear maneuvering systems. This approach is based on the concept of *transverse dynamics*. We have demonstrated the usefulness of this approach in the control of the swinging energy of the pendulum for an *experimental* cart and pendulum system. On the theoretical side, we have provided a new method for the construction of *converse* Lyapunov functions for exponentially stable periodic orbits.

New techniques for the approximate feedback linearization of nonlinear systems have been developed. In order to construct a feedback linearizing coordinate transformation, a class of optimization problems has been formulated for finding approximate solutions to an appropriate system of partial differential equations. In contrast to previous results, this approach *does not* require *differentiation* of the data describing the system and is therefore applicable to systems with, for example, tabular data.

14. SUBJECT TERMS

Nonlinear control.

15. NUMBER OF PAGES

4

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

NSN 7540-01-280-5500

94 3 22 078

DTIC QUALITY INSPECTED 1

Modeling and Control of Nonlinear Systems

AFOSR-91-0255 Final Report
1 May 91 - 31 August 93

John E. Hauser

Dept. of Electrical and Computer Engineering

University of Colorado

Boulder, CO 80309-0425

(303) 492-6496 [fax 492-2758]

hauser@boulder.colorado.edu

work performed at

Department of EE-Systems

University of Southern California

Los Angeles, CA 90089-2563

We have conducted research on the modeling and control of nonlinear systems. Our efforts have been directed toward understanding the control of truly nonlinear behavior as well as the synthesis of control laws for systems that can be nearly transformed into linear systems (via approximate feedback linearization).

Numerical Approximate Feedback Linearization

We have studied the approximate feedback linearization of nonlinear systems of the form

$$\dot{x} = f(x) + g(x)u$$

where f and g are system vector fields that may be available only as numerical data. The problem is to find *coordinate* functions $\phi_i(\cdot)$, $i = 1, \ldots, n$, so that the transformed system ($\xi_i = \phi_i(x)$)

$$\dot{\xi}_1 = \xi_2 \qquad + \psi_1(x) + \theta_1(x)u$$
$$\cdots$$
$$\dot{\xi}_{n-1} = \xi_n \qquad + \psi_{n-1}(x) + \theta_{n-1}(x)u$$
$$\dot{\xi}_n = d(x) + a(x)u \quad + \psi_n(x) + \theta_n(x)u$$

is close to a linear system over some (compact) region Ω of the state space. That is, we wish to make $\psi_i(x) := L_f\phi_i(x) - \phi_{i+1}(x)$ and $\theta_i(x) := L_g\phi_i(x)$ *small* over Ω.

We have developed and implemented algorithms [2, 3] to construct the necessary coordinate functions by solving an unconstrained weighted least squares problem of the form

$$\min_{\phi_i \in \mathcal{F}} \int_{\Omega} \left\{ \sum_{i=1}^{n-1} \left\{ w_f^i(x) \left(\phi_{i+1}(x) - L_f\phi_i(x)\right)^2 + w_g^i(x) \left(L_g\phi_i(x)\right)^2 \right\} \right\}$$

where $\mathcal{F}$ is a suitable finite dimensional function space. In particular, we have selected a space of tensor product B-splines defined over Ω with useful boundary conditions (e.g., $\phi_1(x) = x_1$ and $\phi_i(x) = 0$, $i = 2, \ldots, n$ on the equilibrium manifold $\mathcal{E}$). Noting that $\phi_i(x) = \sum_j B_j(x)\alpha_j^i$ with basis functions $B_j(\cdot)$ this problem then becomes

$$\min_{\alpha} \alpha^T Q \alpha + p^T \alpha + r$$

which can be solved by solving the linear system

$$2Q\alpha = -p.$$

The dimension of the coefficient vector α can become somewhat large depending on the dimension of the space as well as the dimension of the function space. Fortunately, due to the finite support property of B-splines, the matrix Q is sparse. The elements of Q and p are computed by estimating the above integral over the small cubes associated to each basis function. By fitting the components of f and g with polynomials over each of these cubes, we have developed an efficient quadrature-like technique for estimating the elements of Q and p.

We have seen that this approach is effective in constructing feedback linearizable nonlinear system approximations to a given nonlinear system. Unlike most traditional feedback linearization approaches, this new technique *does not* require the differentiation of system vector fields and is thus applicable to systems where the vector fields are only available numerically. Such approximations can be used to develop nonlinear control systems.

Several important questions remain. For example, what is a fair way to compare the *performance* of various nonlinear control schemes? In many cases, it is difficult to characterize what behavior we would like (or should expect) the nonlinear system to exhibit. This is in marked constrast to current state of affairs for linear systems.

Unfortunately, most control objectives continue to be specified from a linear point of view. This, in many cases, leads to quite acceptable results but may be unnecessarily limiting our possibilities. In order to break free from this line of attack, we feel that it is crucial that a number of nonlinear benchmark problems be developed. Furthermore, it is essential that these benchmark problems involve a significant *experimental* component. Without an experimental component, many important nonlinear effects will not be uncovered (we will be merely stretching our *linear* ideas).

Nonlinear Control about a Periodic Orbit

Much of the research in nonlinear control has focused on extending *linear system* theory and results to *nonlinear* systems. However, since linear systems are a strict subset of class of nonlinear systems, this approach will certainly miss many important possibilities. With this in mind, we've been investigating the control of nonlinear systems around period orbits—the simplest truly nonlinear phenomenon. Furthermore, to reduce the prevalence of *artificial* assumptions, we have included a substantial experimental component in our investigation.
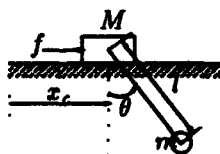

Figure 1: A cart and pendulum system

Our research has been motivated by the common cart and pendulum system shown in Figure 1. Rather than the usual objective of balancing the inverted pendulum, we chose to make regulation of the swinging energy of the pendulum the control objective. The results of this research were presented at the 31st CDC and have been accepted for publication [1].

The (normalized) dynamics of this system have the simple form

$$\begin{aligned} \ddot{\theta} &= -\sin\theta - \cos\theta u \\ \ddot{x}_c &= u \end{aligned} \tag{1}$$

where the input u is taken as the cart acceleration. The objective was to regulate the swing energy $H(\theta, \omega) := \omega^2/2 + (1 - \cos\theta)$ (where $\omega = \dot\theta$) to a desired swing energy $\hat{H}$. Defining the energy error $E(\theta, \omega) := H(\theta, \omega) - \hat{H}$ we chose the control law

$$u = \alpha\omega\cos\theta E - (2\zeta\omega_c v_c + \omega_c^2 x_c) \tag{2}$$

where α, ζ, and ω_c are design parameters and $v_c = \dot{x}_c$. This control law (structure) was selected on the following grounds. If the cart dynamics is completely ignored (in the control and the dynamics), the closed loop error dynamics

$$\dot{E} = -\alpha\omega^2\cos^2\theta E$$

can be shown, using the theory of Poincaré maps, to be exponentially stable (for almost all initial conditions). The remaining portion of the control law was added to stabilize the cart position without destroying the stability of the periodic orbit.

The stability and robustness characteristics of the closed loop system are best understood by looking at the dynamics *transverse* to the desired periodic orbit. For this system, E, x_c, v_c provide suitable transverse coordinates. By showing that the (periodic) time varying linearization of the transverse dynamics about the periodic orbit was asymptotically stable, we were able to conclude that the orbit was itself exponentially stable. Furthermore, since we analyzed the (time-varying) linear system using a small gain approach, the overall system possessed additional robustness properties. These characteristics were further confirmed experimentally. The experimental component also led to interesting insights concerning the use of state estimators for nonlinear control [1].

In general, the local dynamics about a periodic orbit can be described by

$$\begin{aligned} \dot\theta &= 1 + f_1(\theta, \rho) \\ \dot\rho &= A(\theta)\rho + f_2(\theta, \rho) \end{aligned} \tag{3}$$

where f_1 and f_2 are first and second order, respectively, in the *transverse* coordinates ρ. Many powerful analysis and synthesis tools can be contructed by noting that the periodic orbit is exponentially stable if and only if the *transverse linearization*

$$\frac{d\rho}{d\theta} = A(\theta)\rho \tag{4}$$
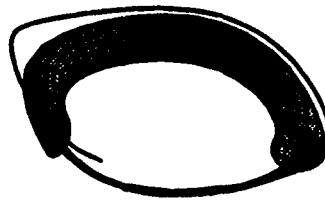
is asymptotically stable.

Figure 2: A domain of attraction estimate for a periodic orbit.

We have, for example, used this fact to provide a means for constructing converse Lyapunov functions for exponentially stable periodic orbits [4]. Such functions can be used to estimate the domain of attraction of the stable periodic orbit. Figure 2 illustrates the domain of attraction of the periodic orbit of a simplified three dimensional version of the closed loop cart and pendulum dynamics (obtained by replacing the second order cart dynamics with a first order dynamics). We are also currently applying the ideas to the nonlinear H_∞ control of systems with periodic orbits.

References

[1] Chung Choo Chung and John Hauser. Nonlinear control of a swinging pendulum. To appear *Automatica*.

[2] Eric C. Gwo and John Hauser. A numerical approach for approximate feedback linearization. *1993 Automatic Control Conference*, San Francisco, CA, 1993.

[3] Eric C. Gwo and John Hauser. Approximate feedback linearization: An L_2 numerical approach. *32nd Conference on Decision and Control*, San Antonio, TX, 1993.

[4] John Hauser and Chung Choo Chung. Converse Lyapunov functions for exponentially stable periodic orbits. To appear *Systems and Control Letters*.